Probability- The Study of Randomness **4.1 and 4.2**

<u>Intro Vocab:</u>

Random (trials)-

- ***individual outcomes are uncertain***
- ***regular distribution in large # of trials***

Probability-

- ***proportion (%) of times the outcome would occur in a large number of trials***

Experimental Probability-

- % of times an event occurred in an expt.
- what DID happen

Ex: If I toss a coin 30 times, and get 12 heads, what the experimental prob. of getting heads?

$$P(H) = 12/30 = 0.4 = 40\%$$

Theoretical Probability-

- % of times an event SHOULD occur

Ex: Using the same coin tossing situation above, whats the theoretical prob. of getting heads?

$$P(H) = 1/2 = 0.5 = 50\%$$

Probability Models-

- *list of all possible outcomes*
- *probability of each outcome*

x	2	3	4	5	....
$P(x)$	$1/36$	$2/36$	$3/36$	$4/36$	....

Sample Space-

- *the set of all possible outcomes*

{red, blue, green, yellow}

$\{2, 3, 4, 5, \ldots, 11, 12\}$

Ex: What is the sample space for the spinner experiment?

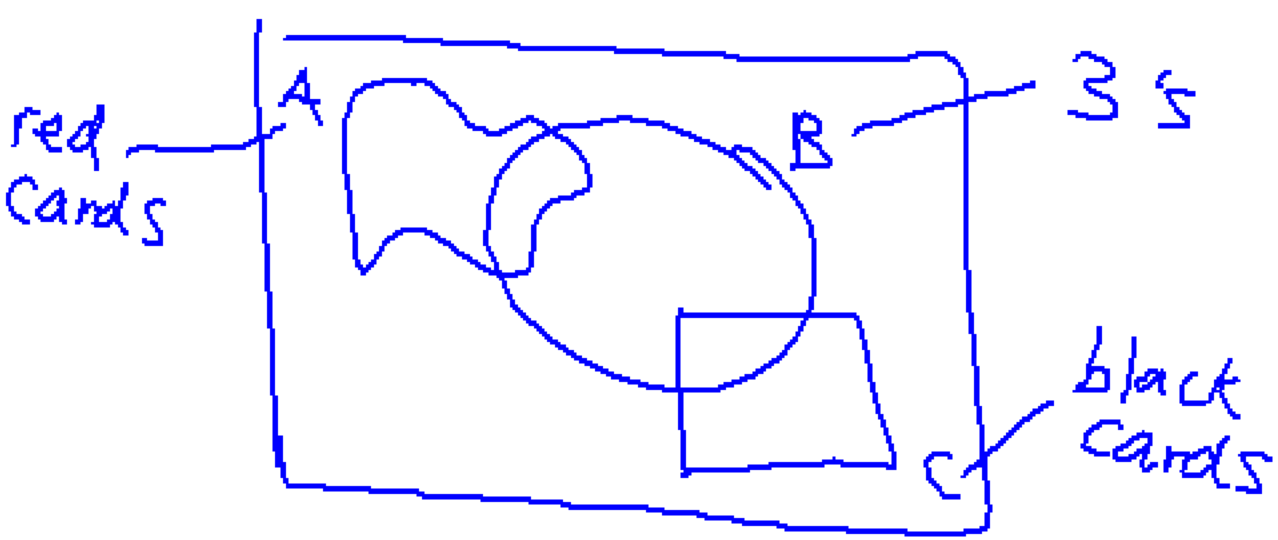
What about the rolling 2 dice experiment?

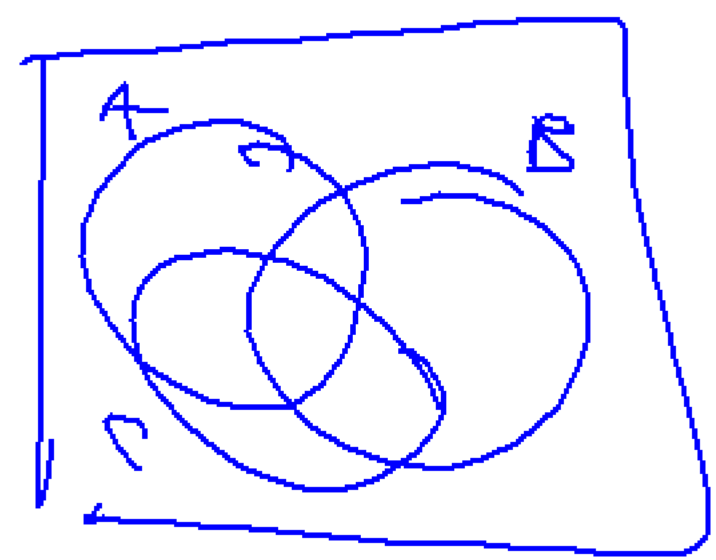
Probability Notation:

- A, B, C, etc. = *events/outcomes*
- P(A) = *the probability of event A occurring*
- S = *sample space*

$P(S)$

- When we represent events, we draw them with *Venn Diagrams*
- Venn Diagrams use *shapes to represent events, sample space is the box around them*
    - Examples: deck of cards

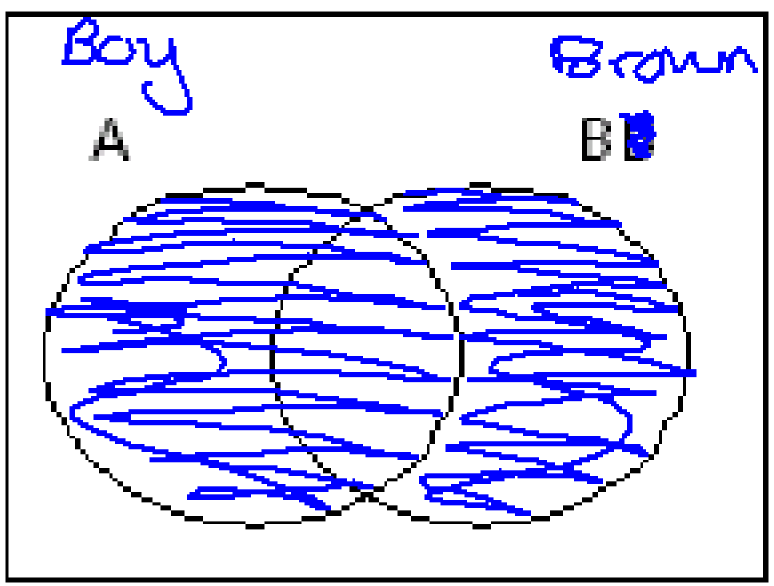




General Set Theory

Union:

- Meaning: join $\boxed{OR}$
- Symbol: $\cup$
- Example 1: $A \cup B$



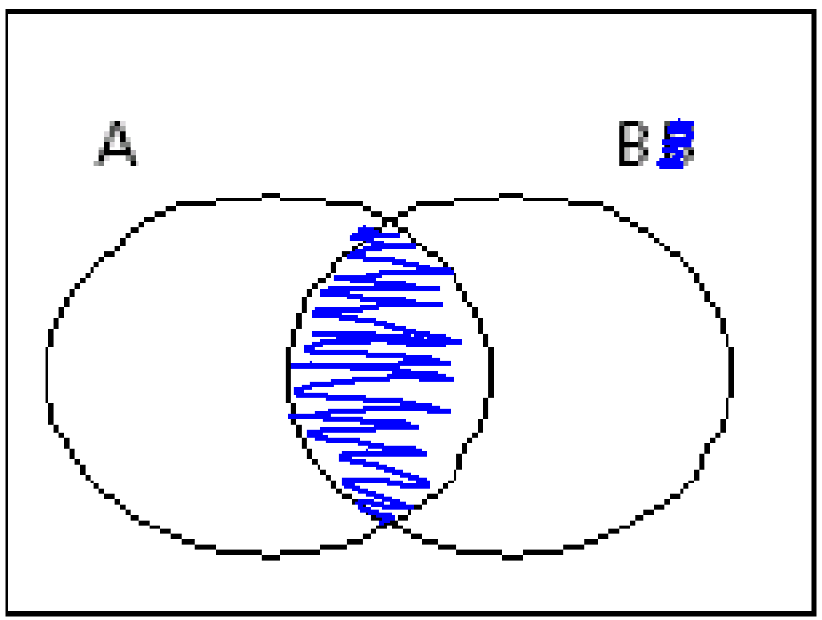
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  A U B = { }

  {1, 2, 3, 4, 5, 6, 7, 8, 10, 12}

Intersection:

- Meaning: overlap AND
- Symbol: $\cap$
- Example 1: $A \cap B$

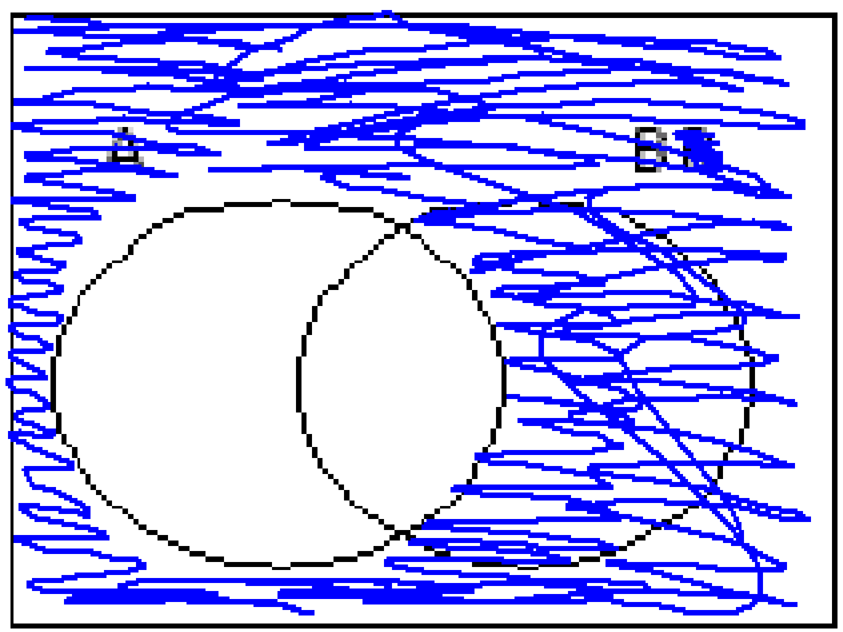


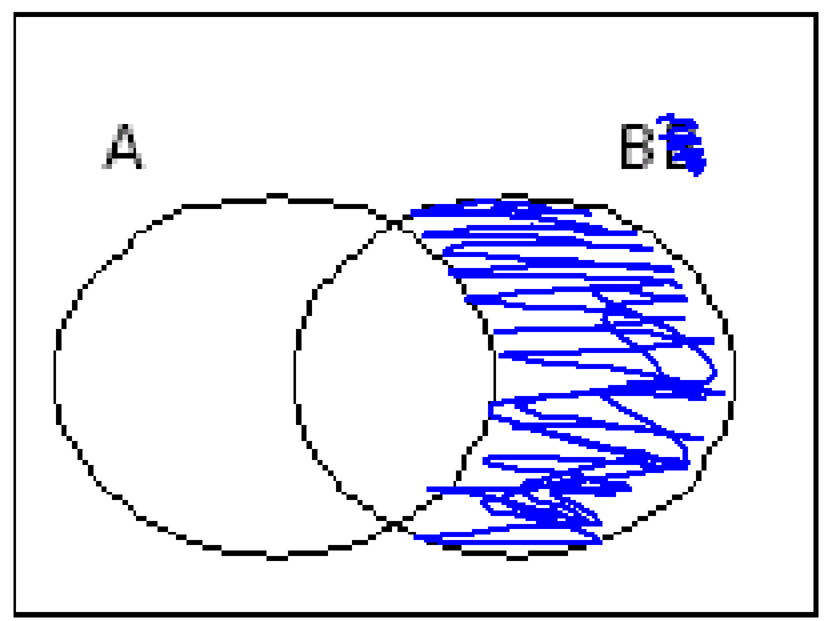
- Example 2: Set A = {2, 4, 6, 8, 10, 12}
  Set B = {1, 2, 3, 4, 5, 6, 7}

  $A \cap B = \{2, 4, 6\}$

Complement:

- Meaning: - the event not occurring
  - everything not in A^c
- Symbol: $A^c = A'$
- Example 1: Shade A^C Shade $A^C \cap B$





- Example 2: Set A = {2, 4, 6, 8, 10, 12}

  S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15} = *sample space*

$A^c = \{ \qquad \}$

{1, 3, 5, 7, 9, 11, 13, 14, 15}

Try the examples:

1) A   B =   **2) P(A   B) =**

3) {1,3,5,6,7,8,9,10,11,13,14,15,17,19,21}
4) 15/25
5) {1,2,4,6,8,10,12,14,16,18,20,22,23,24,25}
6) 15/25
7) {2,4,6,8,10,12,14,16,20,22,24}
8) 11 / 25
9) {∅}   {∅}
10) 0
11) {1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,19,20,21,22,24}
12) 22 / 25
13) {6,8,10,14}
14) 10 / 25
15) { 1,3,5,7,9,11,13,15,17,18,19,21,23,25}
16) 10 / 25

Try the next sheet.... SET THEORY

a) 8 / 20
b) 7 / 20
c) { $\emptyset$ }
d) 0
e) {3,4,5,6,7,8,11,13,15,16,17,22,28,30}
f) 14/ 20
g) 9 / 20
h) 8 / 20
i) 12 / 20
j) 13 / 20
k) 8 / 20
l) 3 / 20
m) 11 / 20
n) 5 / 20
o) 3 / 20
p) 11 / 20
q) 15 / 20
r) 3 / 20

$D^c = \{ \quad \}$

$P(D^c)$

$1 - P(D)$

Probability Rules

- Let A and B be events
- Let S = sample space
- Let Ac = the complement of event A

List the first 3 probability rules: (page 298)

(1) $0 \leq P(A) \leq 1$

(2) $P(S) = 1$

(3) $P(A^c) = 1 - P(A)$

Example 1: If the probability of hitting a homerun is 30%, whats the probability of not hitting a homerun?

$P(H) =$ 0.30

$P(H^C) =$ 0.70

Example 2: If there are only 8 different blood types, fill in the chart below:

Type	A+	A-	B+	B-	AB+	AB-	O+	O-
Probability	0.16	0.14	0.19	0.17	?	0.07	0.1	0.11

0.06

Example 3:

Las Vegas Zeke, when asked to predict the ACC basketball Champion, follows the modern practice of giving probabilistic predictions. He says, "UNC's probability of winning is twice Duke's. NC State and UVA each have probability 0.1 of winning, but Duke's probability is three times that. Nobody else has a chance." Has Zeke given a legitimate assignment of probabilities to all the teams in the conference? Why or why not?

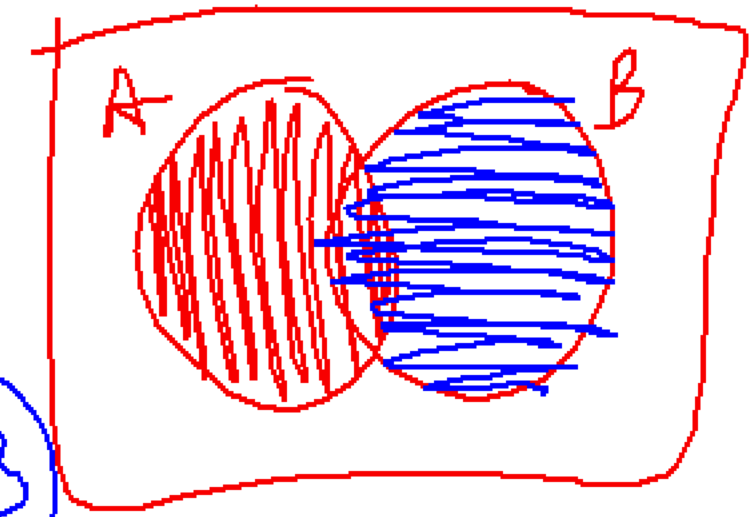
UNC = 0.6

Duke = 0.3

NC State = 0.1

UVA = 0.1

~~1.1~~



Probability Rules (cont'd)

Unions

General Rule:

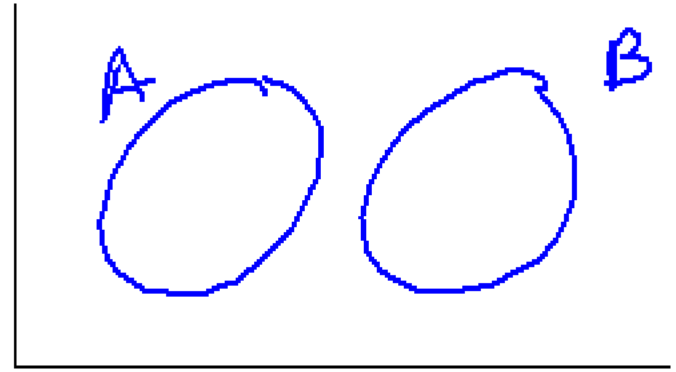
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

  or

- Why do we subtract $P(A \cap B)$?

Special Case:

What if A and B don't overlap? Draw a Venn Diagram that illustrates this below:



So, $P(A \cap B)$ = ? 0

This is called **Disjoint**

Disjoint (or mutually exclusive)= Two events are disjoint if ...

they have no outcomes in common

So our rule for unions for disjoint events then becomes:

- $P(A \cup B) = P(A) + P(B)$

Going back to the examples from before...

Ex #4: There are only 8 different blood types, given in the chart below:

Are these events (the blood types) disjoint? yes

What is the probability of being either Type A+ or B-?

$$P(A+ \cup B-) = P(A+) + P(B-) = 0.33$$

What is the probability of being either Type O- or O+?

$$P(O- \cup O+) = P(O-) + P(O+) = 0.21$$

What is the probability of being either Type AB+ or A+?

$$P(AB+ \cup A+) = 0.22$$

Ex #5: We are picking one card out of a standard 52-card deck (no jokers). The events are the different cards we can pick.

Are these events disjoint? NO

What is the probability of picking a diamond? $P(D) = \frac{13}{52}$

What is the probability of picking a 3? $P(3) = \frac{4}{52}$

What is the probability of picking a diamond and a 3? $P(D \cap 3) = 1/52$

* So, what is the probability of picking a diamond or a 3?

$$P(D \cup 3) = P(D) + P(3) - P(D \cap 3) = 16/52$$

What is the probability of picking a black card?

$$P(B) = 26/52$$

What is the probability of picking a Jack?

$$P(J) = 4/52$$

* What is the probability of picking a black card or a Jack?

$$P(B \cup J) = P(B) + P(J) - P(B \cap J) = 28/52$$

Probability Rules (cont'd)

Intersections AND

EX #6: **Picking poker chips from a bag *with replacement*:**

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

P(Red) = $3/10$ *P(Green) =* $2/10$ *P(Blue) =* $5/10$

What is the probability of picking a red chip and then a blue chip?

$$P(R \cap B) = 3/10 \cdot 5/10 = 15/100$$

What is the probability of picking a green chip and then a red chip?

$$P(G \cap R) = P(G) \cdot P(R) = 2/10 \cdot 3/10 = 6/100$$

What's the probability of picking a blue chip and then a blue chip?

$$P(B \cap B) = 5/10 \cdot 5/10 = 25/100$$

So.... P(A and B) = P(A ∩ B) = $P(A) \cdot P(B)$

But what is the key info that was given in Ex #6?

with replacement

So did the first event happening affect the second event happening? No

This is called...

independent

Other examples of events that are independent:

rolling a die

flip coin

EX #7: Picking poker chips from a bag ***without*** replacement

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

P(Red) = $\frac{3}{10}$ P(Green) = $\frac{2}{10}$ P(Blue) = $\frac{5}{10}$

$P(R) \cdot \cancel{P(B)}$

What is the probability of picking a red and **then** a blue?

$$P(R \cap B) = \frac{3}{10} \cdot \frac{5}{9}$$

$P(R) \cdot P(B \mid R)$

*dependent

What is the probability of picking a blue and then a blue?

$$P(B \cap B) = \frac{5}{10} \cdot \frac{4}{9}$$

What is the probability of picking a green and then a red?

$$P(G \cap R) = \frac{2}{10} \cdot \frac{3}{9}$$

$P(G) \cdot P(R \mid G)$

VOCAB: Conditional Probabilities:

$$P(B|A)$$

given that

A = 1st event that happened
B = 2nd event that happened

So... to find the probability of event A <u>AND</u> event B we....

INTERSECTIONS: * ***P(A and B) = P(A ∩ B) =*** $P(A) \cdot P(B|A)$

special case

IF A and B are INDEPENDENT: ***P(A and B) = P(A ∩ B) =*** $P(A) \cdot P(B)$

Thinking about conditional probabilities…

Does P(B|A) = P(A|B)?

Think about our poker chip example. Picking poker chips from a bag without **replacement**

I have a bag that has 10 poker chips in it. 3 are red, 2 are green, and 5 are blue.

$P(R) = 3/10 \quad P(G) = 2/10 \quad P(B) = 5/10$

What is the probability of picking red given that you picked blue?

$P(R|B) = 3/9$

What is the probability of picking blue given that you picked red?

$P(B|R) = 5/9$

So…

$P(A|B) \neq P(B|A)$

REVIEW:

Union

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

- If A and B are **disjoint**, then **$P(A \cup B) = P(A) + P(B)$** because $P(A \cap B) = 0$

Intersection

$P(A \cap B) = P(A) * P(B|A)$

- If A and B are **independent**, then **$P(A \cap B) = P(A) * P(B)$** because $P(B|A) = P(B)$

Same

Conditional Probability

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad P(A) > 0$$

Try these:

Probability rules worksheet NAME:_______________________

1. If $P(A) = 0.26$ and $P(B) = 0.41$ and $P(A \cap B) = 0.1$, find the following:

   a. $P(A \cup B) =$ 0.57

   b. $P(B|A) =$ 0.385 $P(B|A) = \frac{P(A \cap B)}{P(A)}$

   c. Are A and B disjoint events? Why or why not?

   NO $P(A \cap B) \neq 0$

   d. Are A and B independent events? Why or why not?

   NO $P(B|A) \neq P(B)$

   $P(A) \cdot P(B) \neq P(A \cap B)$

2. If P(A) = 0.42 and P(B) = 0.33 and A and B are independent, what is prob of A and B?

$P(A \cap B) = 0.1386$

3. If P(A) = 0.6 and P(B) = 0.34 and P(B|A) = 0.2, find the following:

a. P(A and B) = $P(A \cap B) = P(A) \cdot P(B|A)$

$\boxed{= 0.12}$

b. P(A or B) =

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \boxed{0.82}$

$P(B|A) = \dfrac{P(A \cap B)}{P(A)}$

$0.2 = \dfrac{P(A \cap B)}{0.6}$

4. Let the sample space, S = {all whole number from 0 through 19}

Let the event A = {2, 4, 6, 8, 10, 12}

Let the event B = {3, 6, 9, 12, 15, 18}

Let the event C = {1, 3, 5, 7, 9, 11, 13, 15, 17, 19}

Let the event D = {1, 4, 7, 8, 10, 14, 16, 18}

Find the following:

a. A B =

b. P(A B) =

c. D^c =

d. P(C B) =

e. P(A U B) =

f. P(C D) =

g. P (C^c) =

h. C U A =

13 $P(E) = 0.42$

$P(E \cap VT) = 0.27$

$P(VT \mid E) = ?$

Probability Rules Review worksheet